4th Grade

Multiplication & Division Relationship

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Multiplication is a fast way of adding a series of repeated numbers

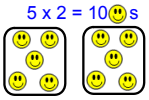
Using repeated addition or skip counting:

5 X 3 means 5 + 5 + 5
5 X 3 = 15

4 X 8 means 4 + 4 + 4 + 4 + 4 + 4 +4 + 4 + 4
4 X 8 = 32

Or you can use a picture model:

5 x 2 = 10 s



Write an example of a multiplication sentence related to addition and solve it.

Place Value and Multiplying

Remember how each place value is 10 x's larger than the next?

If we take any of the place value units, the next unit on the left is ten times as many.

1 ten = 10 x 1 one (1 ten is 10 times as much as 1 one)

1 hundred = 10 x 1 ten

1 thousand = 10 x 1 hundred

Place Value and Multiplying

We can apply this knowledge with larger numbers whose digits are in the tens, hundreds, or even thousands!

10 x 3 = 30
100 x 3 = 300
1,000 x 3 = 3,000

We know these are true because our place value is increasing by 10x's with each equation.

Place Value and Multiplying

We know that if a number is increasing in place value, we need to add the additional place value to our answer.

Let's look at some examples:

4 x 4 = 16	5 x 4 = 20
4 x 40 = 160	50 x 4 = 200
40 x 40 = 1,600	50 x 40 = 2,000

Can you see how the place value is getting larger with each equation? Do you notice a pattern?

It is important to understand how place value works, but there is a TRICK when multiplying numbers that end in zeros!

What is the Trick?

What is 6 x 7 = ?

What if we multiplied 60 x 7 = ?

First, we know the place value increases. You can also use the zero trick by covering up the zero, multiplying 6 x 7, which equals 42, then add the zero back onto your answer.

60 x 7 = 42 → Add zero to end to show 60 x 7 = 420

What is 50 x 2 then?

What about 70 x 80 = ?

Be careful, in this problem you have to cover up 2 zeros, which means you have to add 2 zeros at the end!

Use the Trick

What is 50 x 2 then?

What about 70 x 80 = ?

(Be careful, in this problem you have to cover up 2 zeros, which means you have to add 2 zeros at the end!)

Use the Trick

Try these:

400 x 3 = ___	20 x 50 = ___	8 x 900 = ___
10 x 100 = ___	70 x 40 = ___	300 x 90 = ___

The numbers in a multiplication sentence are represented by factors and the product.

- Factors - numbers you multiply with together to get another number (product)
- Product - the answer when 2 or more numbers are multiplied together

Here are 2 ways to write a multiplication sentence.

2 X 5 = 10 (factor, factor, product)

2(5) = 10 (factor, factor, product)

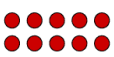
Arrays

Multiplication sentences can also be represented using picture models called arrays.

For example:

2 X 5 means 2 + 2 + 2 + 2 + 2

2 X 5 = 10



4 X 6 means 4 + 4 + 4 + 4 + 4 +4

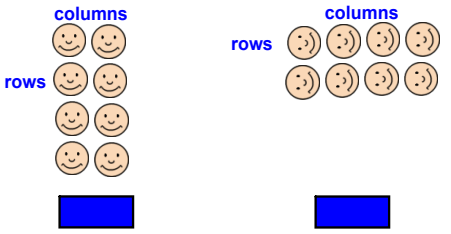
4 X 6 = 24

3 + 3 + 3 + 3 means 3 X 4

3 X 4 = 12

Arrays

Array sentences are written with the number of rows first and the number of columns second.



Create a multiplication sentence, draw a picture to represent your values.

Arrays

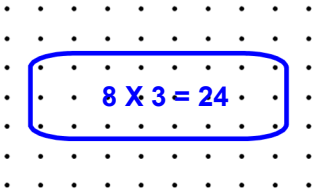
Drag arrows into each rectangle to make the arrays. How are they the same? Different?

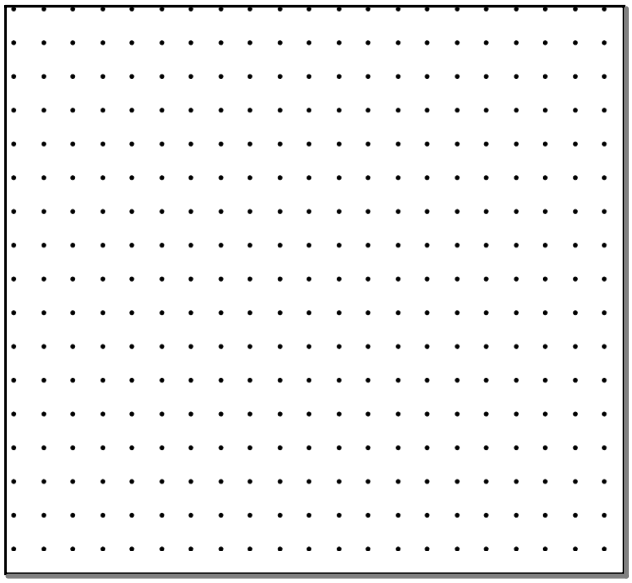
5 x 2	2 x 5

Arrays

1. On a dot paper, draw several arrays
2. Trade your paper with a partner and label your partners arrays with the appropriate multiplication sentence inside or next to the drawing.

Example:





Properties of Multiplication

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Multiplication Properties

They make solving multiplication easier!

1. Zero Property	3 X 0 = 0
2. Identity Property	4 X 1 = 4
3. Commutative Property	5 X 6 = 6 X 5
4. Associative Property	2 X (3 X 4) = 3 X (2 X 4)
5. Distributive Property	9(20 - 3) = (9 X 20) - (9 X 3)

These properties are directly related to the addition properties you learned previously!

Zero Property

Any number multiplied by 0 is **always zero**

0 X 3 = 0 + 0 + 0 = 0

6 X 0 = 0

You can also use variables to represent any value

0 X m = 0

Try this:

If Jackie has 5 hats and zero marbles in each hat. How many marbles does she have in all?

Identity Property

Any number multiplied by ONE is **always the original number**

5 x 1 = 5

1 X 2,345,407 = 2,345,407

Try this:

Solve for p in the following: 234 X p = 234

What tools did you use to find your answer? Tell a partner.

Multiplication Properties

Solve the following equations. Write what multiplication property is represented in all 3 equations, then discuss how you determined the value of the variable in each one.

		Property:
1.	234 X 1 = z	_____
2.	q X 2,567 = 2,567	_____
3.	98,765 X d = 98,765	_____

Commutative Property

The commutative property of multiplication means **the order of the numbers does not change the result (answer)** of the problem

3 X 5 is the same as 5 X 3 (They both equal 15)

hint: To remember this property, think of communicating (talking) with your friends!

Here are some more examples:

a x b = b x a 3 x 8 = 8 x 3

Try This:

How can you finish the equation using the Commutative Property?

7 X 4 = ? X ?

Associative Property

3 X (2 X 4) = 2 X (4 X 3)

(8 X 3) X 9 = 8 X (3 X 9)

4 X (7 X 6) = (4 X 7) X 6

Talk it out: Looking at the examples to the left, how would you define the Associative Property in your own words?

Commutative and Associative Properties

Write the definitions and examples below to the appropriate column.

Commutative	Associative
Keeps the same numerical order, but parenthesis move	Can reorder numbers in the expression
3 X 5 = 5 X 3	5 X (7 X 2) = (5 X 7) X 2

Distributive Property

In the Distributive Property, you distribute, pass, or hand out multiplication to numbers within parenethsis using addition or subtraction.

There are 2 common ways to use this property

Distributive Property

#1: You can use it to find math facts that can be difficult to remember...

Lets solve 6 X 12 = A by distributing 6 into parts of 12

Step 1: Break 12 into easier numbers you can multiply. We know 10 + 2 = 12, right? So...

Step 2: ...if we distribute (pass out) 6 to both digits, we will have (6 X 10) + (6 X 2)

Step 3: Solve the equation starting with multiplication.

6 X 10 = 60 + 6 X 2 = 8

60 + 8 = 68

What is another way you could distribute 12 to solve?

Look in teacher notes for answers

Let's Practice!

How can you solve 8 x 13 by using the Distributive Property?

First, let's think of an easy way to break apart the larger number...

What are possible numbers that add up to equal 13?

Does it make more sense to use 10 + 3 = 13 or 8 + 5 = 13? Why?

Let's use 10 + 3, so applying the Distributive Property to solve would look like this...

8 x 13 = (8 x 10) + (8 x 3)

= 80 + 24

= 104

Derived from engage^ny

Let's Practice!

How can you solve 8 x 13 by using the Distributive Property?

Can you solve 8 x 13, by distributing 13 using the numbers 8 and 5? What is your answer?

Derived from engage^ny

Your Turn

With your elbow partners solve the following using the distributive property. Remember to first decide what 2 numbers make the larger number easier to solve with. For example, in #1, does it make more sense to break 12 into 6 + 6 or 10 + 2? Show your work!

1. 6 x 12 =
2. 34 x 8 =
3. 42 x 4 =

Distributive Property

#2 You can solve an equation with parenthesis by distributing the number on the outside to digits on the inside.

6(9 + 5) = (6 X 9) + (6 X 5)

45 + 30

= 75

Remember a number next to parenthesis means to multiply!

You use addition after you find the products because that is the function inside the parenthesis.

Try this using the distributive property: 9(8 + 6) =

Distributive Property

You can also use it with subtraction. Instead of adding the products together, you will subtract them.

8(7 - 3) = (8 X 7) - (8 X 3)

Notice subtraction being carried throughout since it is function within the parenthesis

56 - 24
= 32

☆ Try this using the distributive property:
3(6 - 4) =

Factors

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What is a FACTOR?

What is the multiplication sentences represented by these arrays?

click to reveal

Both arrays equal the product of 8. Remember, **factors are 2 numbers multiplied to get a given product**. Factors, 1 X 8, and 2 X 4 both multiply to equal the product 8. So we know the factors of 8 are: 1, 2, 4, 8

Derived from engage^ny

What is a FACTOR?

1 X 8 = 8

2 X 4 = 8

You can represent given factors by using a factor rainbow.

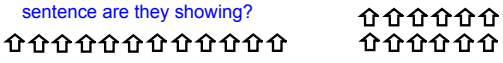
1 2 4 8

Derived from engage^ny

Factor Rainbows

Factor Rainbows help organize the numbers and allow you to check your work to make sure you find ALL factors.

Lets factor 12: Look at the following arrays. What multiplication sentence are they showing?



1 X 12 =12

2 X 6 =12

Circle the factors. Write these factors in numerical order using the factor rainbow.

1 2 6 12

Derived from engage^ny

Factor Rainbows

☆ Draw another array to represent another pair of factors for 12. Now rewrite all the factors found for 12 using a factor rainbow.

click to reveal

Derived from engage^ny

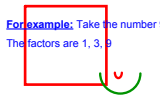
Helpful Hints

1. Always start factoring with the number and 1.
2. Even numbers always have 2 as a factor. You will need to find the number that multiplies with 2 that equals the given number to know the factor pair!
3. Numbers with 5 as a factor have a 0 or 5 in the ones place value.
4. If you make a factor rainbow, and cannot connect a number to another factor it could be....

   - You forgot to find the other factor and should do so.

   - The other factor is the same number.

Helpful Hints



Discuss with your partner why 3 does not have a factor pair, but this factor rainbow is correct.

click to reveal

How do you know you've found ALL the factors?

Lets factor 18:

1. We always start with 1 and the number, in this case 18.

2. Then you continue to think of numbers and/or draw arrays that multiply to represent 18. Work up numerically, going to 2, then 3, and so on. If it doesn't multiply by another factor to equal 18, you know it's not a factor.

3. You will be able to make the factor rainbow, connecting each factor pair when you have found all of the factors!

Factor pairs of 18

1	18
2	9
3	6

Division with Factors

Division can help to find factors of larger numbers

Lets look at the number 54. How can we determine if 3 is a factor of 54?

Use division to determine if 3 is a factor or not: $54 \div 3 = ?$

$$\begin{array}{r} 18 \\ 3\overline{)\,54} \\ -\,3\downarrow \\ \hline 24 \\ -\,24 \\ \hline 0 \end{array}$$

Because there is no remainder, you know that 3 is a factor of 84.

↓

3 and 18 are a factor pair of 54

Use division when unsure of numbers that could be factors!

Division with Factors

How can you find all the factors of 54?

Let's find out together...

1. Begin with 1 and the given number, 54
2. Because 54 is even, we know 2 is a factor

$$\begin{array}{r} 27 \\ 2\overline{)\,54} \\ -\,4\downarrow \\ \hline 14 \\ -\,14 \\ \hline 00 \end{array}$$

2 and 27 are factors

Could you skip count to find the missing factor as well?

3. Previously we found 3 and 18 are factors.
4. Then 4 and so on, until you each 12.

Is it necessary to divide to find if 5 is a factor of 54?

Factors

Using the strategies we just covered, multiplication facts and division, let's find the factor pairs of 60.

Factor 60:

1. We know 1 and 60 are the beginning factors
2. Now we think about the number 2, is 60 an even or add number? It's even so 2 is a factor - now we need to find how many 2s
3. What about 3? Let's use division to find out.

$$\begin{array}{r} 20 \\ 3\overline{)\,60} \\ -\,6 \\ \hline 00 \end{array}$$

4. Now we need to look at 4. Use division again.
5. Now keep working through the digits until you find the rest of the factors .

What are all of the factors?

Factor Pairs	
1	60
2	30
3	20
4	15

Analyzing the Numbers

Is 6 a factor of 54?

If we know 6 is a factor can we say that 2 and 3 are also factors of 54? Is the following multiplication sentence true?

54 = 6 x 9 = (2 x 3) x 9

If we rewrite it vertically we can see how 6 relates to 2 and 3

54 = 6 x 9
= (2 x 3) x 9

Remember the Associative Property? Lets use it to socialize 3 with 9 to check if 2 and 3 are really factors.

54 = 2 x (3 x 9)
54 = 2 x 27 → This proves 2 and 27 are a factor pair of 54
54 = 54

Derived from engage^ny

Associative Property can help us find factors!!!

Let's Practice using Associative Property

Is 6 a factor of 42? How do you know...

We know 6 is factor *click*

Let's use the Associative Property to determine if 2 and 3 are also factors of 42.

42 = 6 x 7
42 = (2 x 3) x 7) Associative Property at work!
42 = 2 (3 x 7)
42 = 2 x 21
42 = 42

We can see that 2 is a factor of 42 because 2 and 21 are a factor pair that multply to equal 42!

Derived from engage^ny

Using Factors with Area

Imagine Suzie and her friend trying to build a sand castle. They want the castle to be 24 square feet when they are done. What are possible side lengths their castle could have?

Using our knowledge of factors and area, we can create different lengths of the sides.

Remember the area formula is length x width → A = L x W

2 x 12 = 24

12
2 [ ]

We know 2 and 12 are a factor pair of 24 so we can make a castle area using these as dimensions.

3 x 8 =24

8
3 [ ]

Suzie could also build her castle 3 by 8 feet.

Using Factors with Area

Dillion needed to build a parking lot for the new high school. They needed it to be 100 square yards total in size. What are 3 possible dimensions Dillion could use to make his parking lot?

Prime and Composite

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Determining Prime/Composite Numbers

Let's Think:

When determining if a number is Prime or Composite, you have to think about the factors.

What do you currently know about factors?

How do you solve for factors?

What do you use to check your work?

Prime Numbers

Let's look: 1 X 7 = 7

What are the factors of this equation?

click

How do you know?

Think about 1 X 5 = 5

What are the factors of this equation?

click

Derived from engage^ny

Prime Numbers

Numbers like 5 and 7 that only have 2 factors, 1 and itself, are called **prime numbers**.

Composite Numbers

Numbers with multiple factors are called **composite numbers**.

Lets look at the number 8:

Factors Pairs of 8	
1	8
2	4

We can see 8 is a **composite number** because it has more than 1 and itself as factors. There are 4 factors of 8. The factor pair 2 and 4 make it a composite number.

Sort the numbers into the columns.

2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Prime	Composite

Exceptions

There are 2 numbers that do not qualify as prime or composite.

0 and 1

0 is not classified by these terms because no matter what number you multiply it by, it is always zero. Therefore, 0 is neither prime or composite.

1 is not classified by these terms because mathematicians have agreed it is easier to define the structure of our number system without it classified. Therefore, 1 is neither prime nor composite.

Derived from engage^ny

111	12	59	54	62	20	78	132
98	18	23	126	81	76	9	54
42	45	41			47	5	139
108	53	109			112	72	83
20	103	14			44	97	29
126	3	36	102	98	123	41	130
138	61	121	98	123	134	127	50
82	11	17	2	97	19	37	110

Circle the prime numbers to help the space shuttle to take off from Earth

Multiples

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What is a Multiple?

When you skip count by any number, the numbers you say are called **multiples**.

Talk it out:
How is a multiple different than a factor?

Let's keep talking:
(small group or partners)

How do we know that 20 is a multiple of 4?

Is 20 a multiple of 5? How do you know?

What about 6? Is 20 a multiple of 6? How do you know?

Derived from engage^ny

Multiples

What are multiples of 8?

click

Challenge question:

We know that 2 X 4 = 8, right? We can reason that 8 is a multiple of 2 and 4 because if we skip count by 2, we get to 8, or if we skip count by 4, we also get to 8.

If 8 is a multiple of 2 and 4, are multiples of 8 also multiples of 2 and 4? Let's find out by looking at 40, a multiple of 8:

40 = (5 X 4) X 2
40 = 20 X 2
Yes, 40 is a multiple of 2. Counting 2 twenty times, gets 40.

40 = 5 X 8
40 = 5 X (4 X 2)
40 = (5 X 2) X 4
40 = 10 X 4
Yes, 40 is a multiple of 4. Counting 4 ten times, gets 40.

Derived from engage^ny

Finding Multiples between 1 - 100

Take the number 84.
Is 84 a multiple of the number 4?

Think about ways you could solve this problem.

When determining if a number is a multiple of another number you use skip counting, or you can also use division. This is very helpful with larger numbers.

$$4\overline{)84}\quad -8\quad 0$$

$$4\overline{)84}\quad -8\quad 04$$

$$4\overline{)84}\quad -8\quad 04\quad -4\quad 0$$

84 is a multiple of 4 because when you divide it out, there are no remainders.

4 X 21 = 84
factor, factor, multiple

Hundreds Chart Activity:

By crossing out multiples of numbers, all of the prime numbers will be identified.

- **Use red to cross out all of the even numbers (2, 4, 6, etc.)**
- **Use green to cross out all of the multiples of 3 (3, 6, 9, etc.) that remain.**
- **Use purple to cross out the multiples of 5 that remain.**
- **Make a list of the remaining numbers. What kind of numbers are they?**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Inverse Operations

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Connecting our Learning

Get with a partner and discuss:

What are 2 math functions you have been repeatedly working with throughout this unit?

click

How do those functions work together when finding factors, multiples, and solving equations?

Think about it

4 X K = 12

8 X 2 = Q

B X 5 = 40

How do you know the answer to each equation?

What math function do you use to solve them?

Connecting our Learning

When solving a given equation or expression, you can use **inverse operations**, to find the solution. I**nverse operations** are the opposite operations that undo each other.

Now look at the examples from the previous page.

Would you change your answer on what operation you use to solve it?

What is different about how you solve the first example to the second one?

4 X K = 12

8 X 2 = Q

B X 5 = 40

Multiplication and division are **inverse operations**. You can use each of them to undo the other in order to solve various equations.

Helpful Hints with Inverse Operations

- Inverse operations are used to solve unknowns in an equation.
- An unknown can be represented using a ☐, ?, or a letter to stand for the missing number.
- A letter that stands for a missing number in an equation is called a variable.
- Multiplication and division are inverse operations.
- Addition and subtraction are inverse operations.

Inverse Operations

Take the algebraic expression: 2m = 14

(Remember 2m means to multiply, 2 times the amount of "m".)

Let's rewrite it so we see the multiplication sign:

2 x m = 14

Now, we need to "move" the 2 to the right side of the equation by dividing, which is the inverse operation of multiplication.

$\frac{2 \times m}{2} = \frac{14}{2} \longrightarrow m = 14 \div 2$

The last step is to solve.

m = 7, because 14 divided by 2 equals 7.

Fact Families Use Inverse Operations

Fact Families are an easy way to use inverse operations. Take the numbers, 4, 7, and 28. These numbers create a fact family using multiplication and division.

7 x 4 = 28

4 X 7 = 28

28 ÷ 7 = 4

28 ÷ 4 = 7

is the division that undoes the multiplication of 7 x 4

Try this: 72 ÷ 8 = 9

What inverse operation can you use to undo this equation?

Write the new equation. Is there more than one way to write it?

Inverse Operations

Draw a line to match each equation with its inverse.

$24 \div 6 = 4$

$35 \div 7 = 5$

$60 \div 10 = 6$

$7 \times 5 = 35$

$6 \times 10 = 60$

$24 \div 3 = 8$

$4 \times 6 = 24$

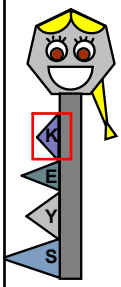
$8 \times 3 = 24$

Working out Word Problems

Using inverse operations to help solve word problems is an important tool to learn.

KAYLEE the Key will help guide you through the steps of problem solving.

The **K.E.Y.S.** to Problem Solving



K: Know the important information in the problem.
Read the problem (more than once) and first find the main idea.
(MAIN IDEA = What is the problem asking you to find?)
Find all the important information that supports the main idea.

E: Equation (or equations) is created to plan your strategy and organize the important information.
Use equations to develop a strategy (i.e. algorithm, diagram).
Strategy must be organized and easy to follow.

Y: Yes, I have checked over my strategy and my answer is reasonable (makes sense).
Use an estimate to check if your answer is reasonable.

S: Solution is written in with the correct label and explanation if needed.

Inverse Operations Word Problems

The animal shelter has 18 kittens. The same number of kittens were born to each of 3 mother cats. How many kittens did each mother cat have?

Pull out the information you need to solve.

Write the equation to solve the unknown value:

Solve

Write your solution:

Inverse Operations Word Problems

The cost of buying a movie is 4 times as much as renting one. It costs $20 to buy a movie. What is the cost, in dollars, of renting a movie?

Pull out the information you need to solve.

Write the equation to solve the unknown value:

Solve

Write your solution:

Inverse Operations Word Problems

Try this: With a partner read the following, write an equation, and solve using inverse operations.

A new video game you want is $42. How much money do you need to save per week if you want to buy it in 7 weeks.

Inverse Operations Word Problems

Try this: With a partner read the following, write an equation, and solve using inverse operations.

Stan was playing a game with this friend. His friend was trying to guess the number he was thinking of. Stan told him if he multiplied his number by 2 and added 9 the answer would be 21. Write an equation. Then use inverse operations to solve.